21/11/2015 السبت د. سيد سلام محاضرة $\boxed{8}$

remember : ${}^nC_r = {}^nC_{n-r}$

Example :

If a coin is tossed five times, assuming that the probability of getting a head in any trial is 1/3, find the probability of getting

i) exactly 3 heads ii) at least 3 heads

iii) at most 3 heads

i) $3h \Rightarrow P(3; 5, \frac{1}{3}) = \left({}^5C_3\right) p^3 q^2$

$= \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \times \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 = 10 \left(\frac{1}{3}\right)^5 . 4$

$= \frac{40}{243}$

ii) at least $3h \Rightarrow P = P(3h) + P(4h) + P(5h)$

$= \binom{5}{3} p^3 q^2 + \binom{5}{4} p^4 q^1 + \binom{5}{5} p^5 q^0$

$= \frac{5 . 4 . 3}{3 . 2 . 1} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 + 5 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) + 1 \left(\frac{1}{3}\right)^5$

$= \frac{40}{243} + \frac{10}{243} + \frac{1}{243} = \frac{51}{243}$

iii) at most $3h \Rightarrow P = P(3h) + P(2h) + P(h) + P(0)$

$= \frac{40}{243} + \binom{5}{2} p^2 q^3 + \binom{5}{1} p^1 q^4 + \binom{5}{0} p^0 q^5$

$= \frac{40}{243} + \frac{5.4}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3 + 5 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$

$+ \left(\frac{2}{3}\right)^5 = \frac{1}{243} \left[40 + 80 + 80 + 32\right.$

$= \frac{232}{243}$

anlogous to the previous example:-

i) exactly $2T = P(2;5,\frac{2}{3})$

$$= \binom{2}{5}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 = \frac{40}{243}$$

ii) almost $2T = P(2T) + P(1T) + P(0)$

$$= \ldots + \ldots + \ldots = \frac{51}{243}$$

iii) at least $2T = P(2T) + P(3T) + P(4T)$

$$+ P(5T) = \frac{232}{243}$$

Properties of the binomial distribution:-

Let X be a random variable with the binomial distribution $P(K;n,p)$, then

i- Mean $= \mu = m = np$

ii- Variance $= \sigma^2 = npq$

iii- Standard deviation $= \sigma = \sqrt{npq} = S.D.$

$$\text{mean} = E(x) = \sum_{0}^{n} x \cdot p(x)$$

$$= \sum_{K=0}^{n} K \cdot P(K;n,p) = 0 \times \frac{n!}{0!\,(n-0)!} p^0 q^{n-0} +$$

$$+ \sum_{K=1}^{n} \frac{n!}{K!\,(n-K)!} p^K q^{n-K}$$

$$= \sum_{K=1}^{n} \frac{n(n-K)}{K(K-1)!\,(n-K)!} p \cdot p^{K-1} q^{n-K}$$

$$= np \sum_{K=1}^{n} \frac{(n-1)!}{(K-1)!\,(n-K)!} p^{K-1} q^{(n-1-K+1)}$$

Let $S = K - 1$, and $N = n - 1$

as K runs through the value 1 to n, S runs through the values 0 to N

$S = K - 1$ at $K = 1$ $\therefore S = 0$

at $K = n$ $\therefore S = n - 1 = N$

we denote that we drop the term $K = 0$, since its value is zero and we factor out np from each term

$$E(x) = np \sum_{S=0}^{N} \frac{N!}{S!\,[(n-1)-(K-1)]} p^S q^{n-S}$$

$$= np \sum_{S=0}^{N} \frac{N!}{S!\,[N-S]!} p^S q^{N-S} = np \sum_{S=0}^{N} \binom{N}{S} p^S q^{N-S}$$

$$= np\,(\underbrace{p+q}_{1})^N = np$$

Poisson Distribution :-

The probability of Poisson Distribution is defined as

$$P(K;\lambda) = \frac{\lambda^K e^{-\lambda}}{K!}, \quad K = 0, 1, 2, \ldots$$

where λ is some Constant.

Poisson distribution can be regarded as the Limit of the binomial distribution when the trials n becomes very Large ($n \to \infty$) and the probability of success become very small. But np remains a finite Constant λ, $q = 1 - P$.

This countably ifinite distribution appears in many natural phenomena, such as the number of telephone calls per minute at some switch board, the number of misprints per page in a large text, and the number of α particles emitted by a radio active substance.

Properties of poisson distribution

mean $= \mu = \lambda$

Variance $= \sigma^2 = \lambda$

S.D. $= = \sigma = \sqrt{\lambda}$

example:

For Poisson distribution, find:

i - $P(2;1) = \dfrac{1 * e^{-1}}{2!} = 0.184$

ii - $p(3, \frac{1}{2}) = \dfrac{(1/3)^3 \cdot e^{-0.5}}{3!} = 0.013$

iii - $p(2, 0.7) = \dfrac{(0.7)^2 \cdot e^{-0.7}}{2!} = 0.12$